Viewing the world systemically.

ATIS: Connected Components & Affect Relation Properties

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The SimEd Basic Logic as Founded on the
Logic of Axiomatic-General Systems Behavioral Theory:

Connected Components and Affect Relation Properties

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Introduction

Affect relations define the relation-set of a General System. In SIGGS these relations were defined in terms of graph theoretic properties. For *ATIS* the graph theoretic properties of SIGGS will be extended to provide greater refinement of the properties and to make them more precise.

In addition to greater precision, a logical analysis of these "connected properties" will help us to recognize additional properties that were not considered in SIGGS.

Further, affect relations that are empirically determined will be used to define the object-set of the general system. This will be made possible by a proper definition of 'affect relation'. All affect relations of *ATIS* will be defined so that they will have the following form: $\{\{x\}, \{x,y\}\}$. This form clearly defines the ordered pair, (x,y), where the direction of the affect relation is from x to y. If the direction of the relation is unknown, then both $\{\{x\}, \{x,y\}\}$and $\{\{y\}, \{x,y\}\}$will be considered elements of the relation-set.

Since in *ATIS* we are concerned with systems that have more than one relation, the General System relation-set is actually a family of relations and will be designated by '$\mathcal{A}$' with elements designated as follows: $\mathcal{A}_n \in \mathcal{A}$. Then, the elements, or components, of each affect relation take the following form: $\{\{x_i\},\{x_i,y_i\}\} \in \mathcal{A}_i \in \mathcal{A}$.

Certain functions will be required to facilitate the analysis of relations in *ATIS*. These functions are: μ, β, φ, and η. These functions are defined as follows:

$$\mu\mathcal{A}_i = \{x_i\};$$

$$\beta\mathcal{A}_i = \{x_i,y_i\};$$

$$\varphi\{x_i,y_i\} = y_i = \varphi\circ\beta\mathcal{A}_i; \text{ and}$$

$$\eta\mathcal{A}_i = \mu\mathcal{A}_i \cap \beta\mathcal{A}_i = x_i.$$

The functions φ and η make it possible to define the elements of the object-set from the elements of the relation set. Whereas the elements of the relation-set are normally defined following the identification of the object-set, for *ATIS*, it is frequently easier to identify the relations empirically before the corresponding object-set elements are actually known. The relation-set elements thereby define the object-set elements. In fact, it is impossible to actually define an object, or component, without first identifying a relation.

If the affect relations define a General System, then a decision-procedure must be determined whereby the affect relations are known. Affect relations will be defined by *Predicate Schemas*, $P(x_n, y_n) = P(\mathcal{A}_n)$. The families of affect relations, $\mathcal{A}$, are defined as extensions of the predicate schemas; that is, $\mathcal{A}_n \in \mathcal{A}$. '$P(\mathcal{A}_n)$' designates the predicate that defines $\mathcal{A}_n$.

Graph-Theoretic Properties

Affect relations for both SIGGS and *ATIS* have their foundation in Graph Theory. In this section we will present the *Graph Theoretic* concepts that will be of value in defining affect relations.

Affect Relation Properties will be defined in terms of path-connected elements, ${}_{pc}\boldsymbol{E}$.

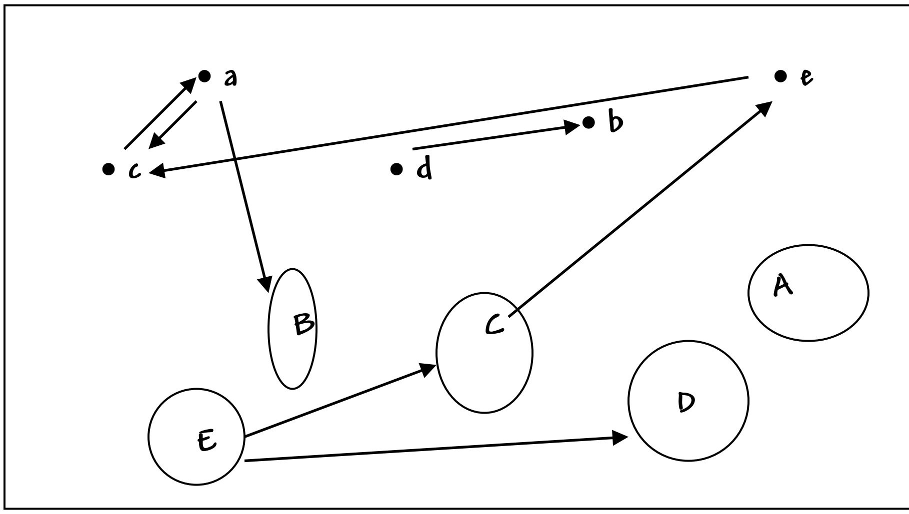
Path-connected elements, ${}_{pc}\boldsymbol{E}$, =$_{\text{df}}$

$$\{(x,y) | (x = x_0, x_1, x_2, \ldots x_{n-1}, x_n = y) \wedge \forall (x_i, y_i)_{i<n}[y_i = x_{i+1}]\}$$

Path-connectedness is intuitively defined as the ability to get from one element to another by following a sequence of elements. The connected paths are "channels" in terms of information theory, or "communications" between the elements of a system. In terms of physics, they are the empirical connectedness that is observed. These are graph-theoretic properties that will be used to define General System properties.

Additional concepts and properties are presented below.

The diagram shown below in Figure 1 depicts the path-connectedness of elements a, b, c, d, and e; and the path-connectedness of sets, A, B, C, D, and E.



The following component path-connections are depicted: (a,c), (c,a), (e,c), (e,a), (d,b).

The following set path-connections are depicted: (E,B), (E,C), and (E,D).

The following component and set path-connections are depicted: (a,B), (C,e), (C,c), (C,a), (e,B), (E,e), (E,c), (E,a).

Figure 1. Path-connectedness of elements (objects, components) and sets.

Discrete segment and *segment cardinality* are defined as follows:

Discrete segment, $|(x,y)_{n=1}|$ = 1, $=_{df}$ A path between two and only two elements.

$$|(x,y)_{n=1}| = 1 = |\{(x,y) \mid (x = x_0, y = x_1)\}|.$$

For example, in Figure 1, (a,c), (c,a), (e,c), and (d,b) are ***discrete segments***.

Segment cardinality, $|(x,y)_n|$ = n, $=_{df}$ The number of discrete segments between elements.

$$|(x,y)_n| = n = |\{(x,y) \mid (x = x_0, x_1, x_2, \ldots x_{n-1}, x_n = y)\}|.$$

For example, in Figure 1, |(a,c)| = 1, |(e,c)| = 2, and |(E,a)| = 4.

The following diagram and symbol conventions will be used to clarify and define the graph-theoretic properties.

Arrows (→, ↔, ←) will be used to show direction of an affect relation between elements (objects, components) of a system.

'(p,q)' designates the **connected elements** p and q and will be considered as an "ordered pair" from p to q unless stated otherwise or the context determines otherwise.

'p → q' designates the ordered pair **path-connected elements** from p to q.

The following diagram, in addition to helping to clarify the connectedness properties, will also be used to introduce terminology that is useful for describing connectedness.

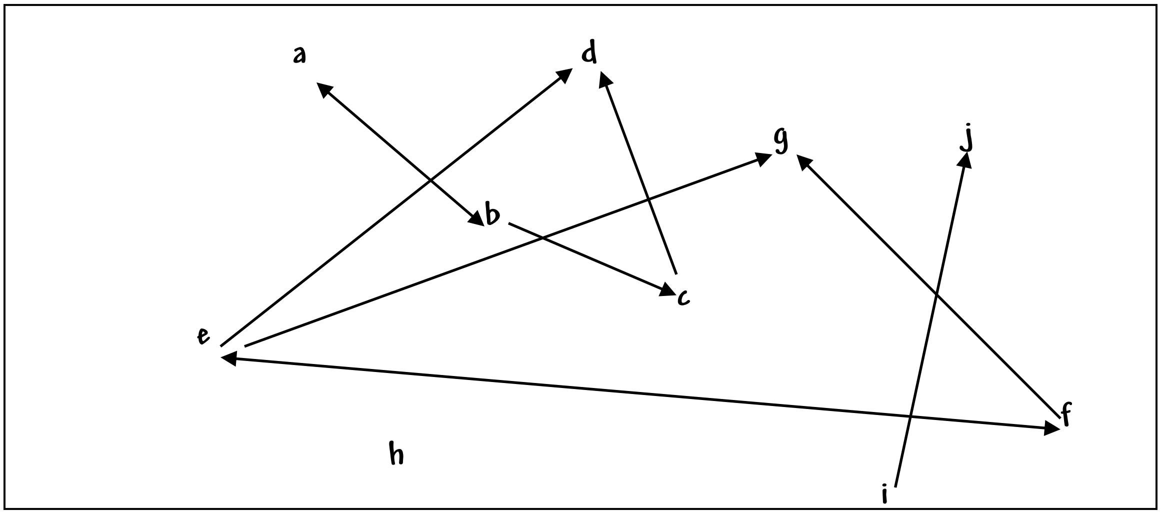


Figure 2. Diagram of Directed-Components Connectedness.

The following list is presented to facilitate the understanding of the various connectedness relationships. From the above graph in Figure 2, the following relations are determined:

Path-connected elements:
(a,b), (b,a), (a,c), (a,d), (b,c), (b,d), (c,d), (e,d), (e,f), (f,d), (f,e), (f,g), and (i,j).

Path-connected elements with three segments: (a,d).

Connected elements: (all path-connected elements) + (a,e), (a,f), (a,g), (b,e), (b,f), (b,g), (c,e), (c,f), and (c,g).

Completely-connected elements: (a,b) and (e,f).

Unilaterally-connected elements: (a,c), (a,d), (b,c), (b,d), (e,d), (e,g), (f,g), and (i,j).

Disconnected elements: All h-pairs of elements, and all i and j pairs except for (i,j).

Receiving elements: a, b, c, d, e, f, g, and j.

Initiating elements: a, b, c, e, f, and i.

Primary-initiating elements: i; that is, it initiates, but does not receive.

h may be considered as a trivial **primary initiating element**.

Terminating elements: d, g, and j.

h may be considered as a trivial **terminating element**.

All terminating elements must be **unilaterally terminating elements**.

The terms described above will be formally defined below. **Path-connected elements** will be restated so as to bring all of the graph-theoretic properties together in one listing.

Path-connected elements, $_{pc}\boldsymbol{E}$, $=_{\mathrm{df}} \{(x,y)|\ (x = x_0, x_1, x_2, \ldots x_{n-1}, x_n = y) \wedge \forall(x_i,y_i)_{i<n}[y_i = x_{i+1}]\}$

Connected elements, $_{c}\boldsymbol{E}$, $=_{\mathrm{df}} \{(x,y)|\ (x = x_0, x_1, x_2, \ldots x_{n-1}, x_n = y) \vee \forall(x_i,y_i)((x_i,y_i)\in{}_{pc}\boldsymbol{E} \vee (y_i,x_i)\in{}_{pc}\boldsymbol{E})\}$

Completely connected elements, $_{cc}\boldsymbol{E}$, $=_{\mathrm{df}} \{(x,y)|\ \forall(x,y)[(x,y),\ (y,x)\in{}_{pc}\boldsymbol{E}]\}$

Unilaterally connected elements, $_{uc}\boldsymbol{E}$, $=_{\mathrm{df}} \{(x,y)|\ \forall(x,y)[(x,y)\in{}_{pc}\boldsymbol{E}.\wedge.\ (y,x)\notin{}_{pc}\boldsymbol{E}]\}$

Initiating elements, $_{i}\boldsymbol{E}$, $=_{\mathrm{df}} \{x|\ \forall x[(x,y)\in{}_{pc}\boldsymbol{E}]\}$

Primary-initiating elements, $_{pi}\boldsymbol{E}$, $=_{\mathrm{df}} \{x|\ \exists y[(x,y)\in{}_{pc}\boldsymbol{E} \wedge \forall u(u,x)\notin{}_{pc}\boldsymbol{E}]\}$

Receiving elements, $_{r}\boldsymbol{E}$, $=_{\mathrm{df}} \{y|\ \forall y[(x,y)\in{}_{pc}\boldsymbol{E}]\}$

Terminating elements, $_{t}\boldsymbol{E}$, $=_{\mathrm{df}} \{y|\ \forall y[(x,y)\in{}_{pc}\boldsymbol{E} \wedge \forall u(y,u)\notin{}_{pc}\boldsymbol{E}\}$

Disconnected elements, $_{d}\boldsymbol{E}$, $=_{\mathrm{df}} \{x|\ \forall(x,y)[(x,y),(y,x)\notin{}_{pc}\boldsymbol{E}]\}$

The distinction must be made between **component-connected properties** and **system-connected properties**.

Component-connected properties describe relations between components; for example, that two components are unilaterally connected. Component-connected properties are defined by the graph theoretic properties of the components.

System-connected properties describe the characteristic pattern of all components of the system with respect to a specific component property; for example, the unilateral connections of the system components are such that the system is characterized by *strongness*. Affect relations define system-connected properties.

Affect Relation Properties

The properties and concepts developed above will now be used to define affect relation system properties for *ATIS*. '$\mathscr{C}$' designates *Component* connected properties. Component connected properties can be measured. The measure, $\mathscr{M}$, of component-connected properties will be defined by $\log_2$ of the cardinality of the set of connected components divided by $\log_2$ of the object-set—see definition below. Consider the following component set and its subset.

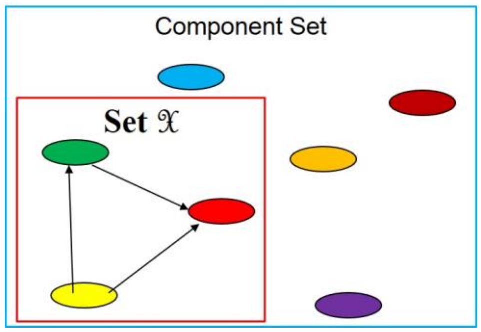


Figure 3. Diagram of a component set with 7 components which has a subset, $\mathfrak{X}$, of 3 connected components. Since the Component Set has 7 components, all of the components must be connected in some way. However, the components in set $\mathfrak{X}$ have different affect relation connections than those in the rest of the Components Set. But, all 7 components must be connected in some way in order to be recognized as being within the Component Set. For example, this set $\mathfrak{X}$ may have connected relations, "Instructs". Here, "Green instructs Red", "Yellow instructs Red and Green", and Red does not instruct anyone. $|(\mathfrak{X})| = 3$; that is, the cardinality of $\mathfrak{X} = 3$.

Evaluating the measure of the connected set, as defined below, we have the following:
$\log_2(|\mathfrak{X}|) \div \log_2(|\mathbb{S}_0|) = \log_2(3) \div \log_2(7) = 1.585 \div 2.807 = 0.565$.

That is, in general, we have the following measure:

$$\mathscr{M}_{\mathscr{C}}(\mathfrak{X}) =_{\text{df}} \log_2(|\mathfrak{X}|) \div \log_2(|\mathbb{S}_0|) = \nu;\ \text{where “}\mathscr{M}_{\mathscr{C}}\text{” is } \textit{component measure}.$$

Connected components, ${}_{C}\mathscr{C}$, $=_{\text{df}}$ a set of system components that are connected to one or more other components.

$${}_{C}\mathscr{C} =_{\text{df}} \mathfrak{X} = \{x \mid x \in \mathbb{S}_0 \wedge \exists y(y \in \mathbb{S}_0 \wedge (x,y) \in {}_{c}\boldsymbol{E} \wedge x \neq y)\}$$

Connected components are defined as components of the object set; such that, the components are connected to other non-equal components.

NOTE: For now we will be concerned only with path-connected components. However, as seen by the definition of **Connected Components**, components may be connected even if there is not a path from one to the other. Further, it may be that we know that two or more components are connected but we do not know the direction of that connection and, therefore, cannot establish the path between them. Such connections will eventually be considered, but, for now, it is easier to visualize connections that are path-connected.

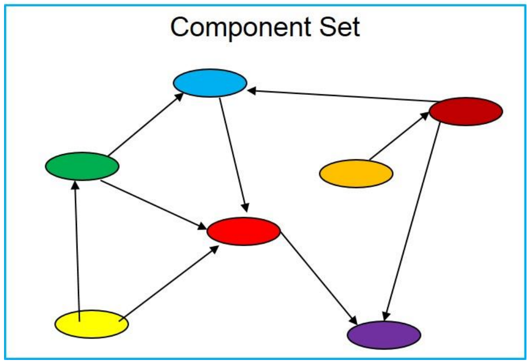


Figure 4. Diagram of a set of **connected components**.

If all of these connections represent the affect relation "Instructs", then the Component Set is a **connected affect relation**, ${}_c\mathcal{A}$, (defined below) and will be designated: ${}_c\mathcal{A}^{\text{instructs}}$.

Connected affect relation, ${}_c\boldsymbol{\mathcal{A}}$, $=_{df}$ components that are similarly connected; that is, they satisfy the same predicate.

$$ {}_c\boldsymbol{\mathcal{A}} =_{df} \mathfrak{X} = \{\{\{x\},\{x,y\}\} \mid P(x,y) \wedge x \neq y \wedge x \in {}_C\mathscr{C}\} $$

Connected affect relation is defined as a set of connected components; such that, the components are distinct and satisfy a given predicate.

For example, in Figure 4, the predicate is: $P(x,y)$ = "x instructs y".

Completely-connected components, ${}_{CC}\mathscr{C}$, $=_{df}$ a set of system components that are pair-wise path-connected in both directions.

$$ {}_{CC}\mathscr{C} =_{df} \mathfrak{X} = \{x \mid x \in \mathfrak{S}_0 \wedge \exists y[(x,y) \in {}_c\boldsymbol{E} \supset (x,y) \in {}_{cc}\boldsymbol{E}]\} $$

Completely-connected components is defined as a set of components of the object set; such that, if the components are connected, then they are completely connected.

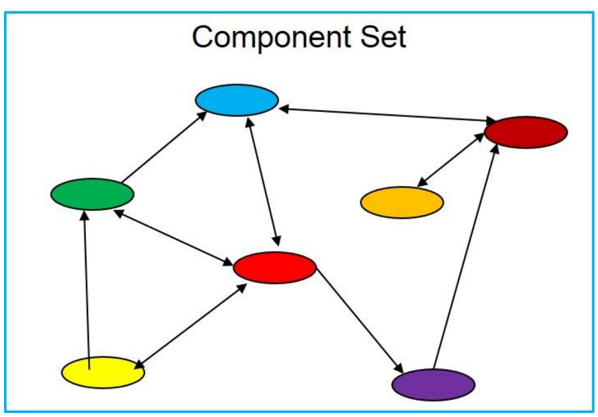


Figure 5. Diagram of a set of **completely-connected components**. That is, from every component there is a "path" (follow thee arrows) to every other component.

Therefore, this component set defines a **completely-connected affect relation** (see definition below) when the connections are defined by the same affect relation.

Completely-connected affect relation, ${}_{CC}\mathcal{A}$, $=_{df}$ an affect relation comprised only of completely connected components.

$$ {}_{CC}\mathcal{A} =_{df} \mathcal{A} \mid \forall(x,y)\in\mathcal{A}\,[x\in {}_{CC}\mathcal{C}] $$

Completely connected affect relation is defined as an affect relation; such that, all components are completely connected.

Direct-directed-connected components, ${}_{DD}\mathcal{C}$, $=_{df}$ a set of system components that are connected with a single directed-path.

$$ {}_{DD}\mathcal{C} =_{df} \mathfrak{X} = \{x \mid x\in \mathfrak{S}_0 \wedge \exists y[(x,y)\in {}_{c}\boldsymbol{E} \supset (x,y)_{n=1}\in {}_{pc}\boldsymbol{E})]\} $$

Direct-directed-connected components is defined as a set of components of the object set; such that, if the components are connected, then they are path-connected by a single directed-path.

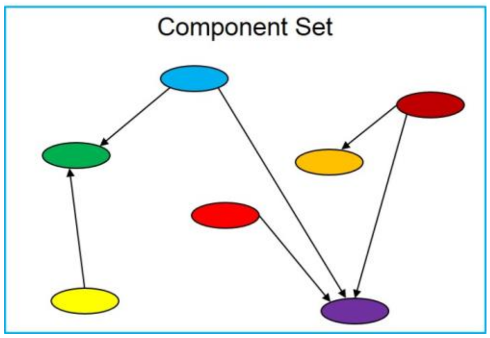


Figure 6. Diagram of a set of **direct-directed-connected components**. That is, if two components are connected, then they are connected by a single path.

Therefore, this component set defines a **direct-directed affect relation** (see definition below) when the connections are defined by the same affect relation.

NOTE: If the connections are not of the same affect relation, then they are not part of the same set.

Direct-directed affect relation, ${}_{DD}\mathcal{A}$, $=_{df}$ an affect relation comprised of direct-directed components.

$$ {}_{DD}\mathcal{A} =_{df} \mathcal{A} \mid \forall(x,y)\in\mathcal{A}\,[x\in {}_{DD}\mathcal{C}] $$

Direct-directed affect relation is defined as an affect relation; such that, all components are direct-directed connected components.

Directed-connected components, ${}_{D}\mathcal{C}$, $=_{df}$ a set of system components that are path-connected.

$$ {}_{D}\mathcal{C} =_{df} \mathfrak{X} = \{x \mid x\in \mathfrak{S}_0 \wedge \exists y[(x,y)\in {}_{c}\boldsymbol{E} \supset (x,y)\in {}_{pc}\boldsymbol{E}]\} $$

Directed-connected components is defined as a set of components of the object set; such that, if the components are connected, then they are path-connected.

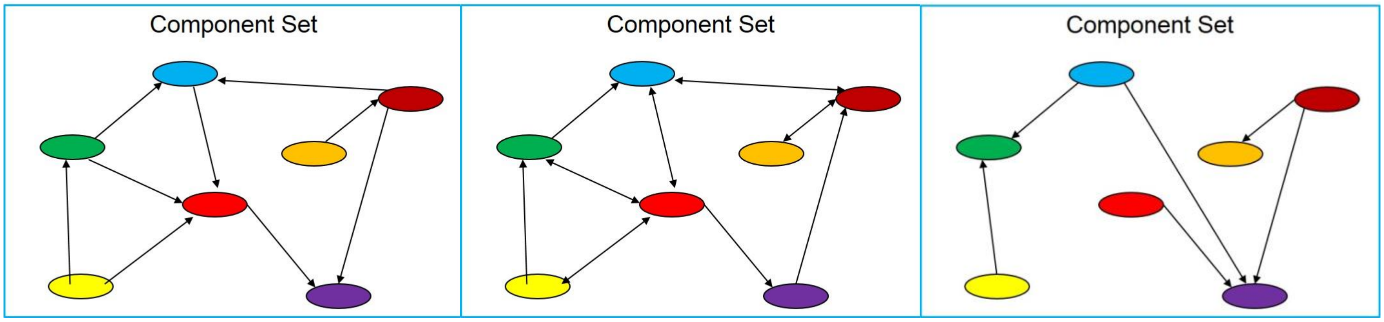


Figure 7. Diagrams of sets that are **directed components**. These are three of the figures that were presented previously. In each case, for any components that are connected, then they are path-connected as demonstrated by the direction of the arrows.

Therefore, these component sets define **directed affect relations** (see definition below) when the connections are defined by the same affect relation. Each component set can represent a different affect relation.

Directed affect relation, $_{D}\mathcal{A}$, $=_{df}$ an affect relation comprised of directed components.

$$_{DD}\mathcal{A} =_{df} \mathcal{A} \mid \forall(x,y)\in\mathcal{A}\,[x\in {}_{D}\mathcal{C}]$$

Directed affect relation is defined as an affect relation; such that, all components are direct directed connected components.

Disconnected components, $_{\sim}\mathcal{C}$, $=_{df}$ a set of system components that are not connected to any other component.

$$_{\sim}\mathcal{C} =_{df} \mathfrak{X} = \{x \mid x\in \mathbb{S}_0 \wedge \forall y[y\in \mathbb{S}_0 \supset (x,y)\notin {}_{c}\boldsymbol{E})]\}$$

Disconnected components is defined as a set of components of the object-set; such that, the components are not connected to any other component of the object-set.

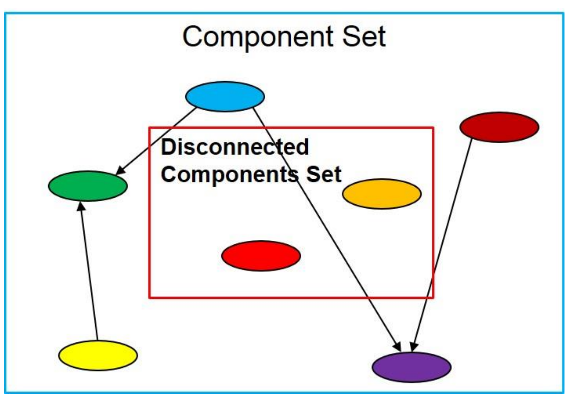


Figure 8. Diagram of a set of **disconnected components**. The Red and Orange components are disconnected with respect to the rest of the components defined by an affect relation. However, in order for them to be even recognized within a system defined by the Component Set, they do have to have some affect relation defined between them and some of the other components. However, for the defined affect relation, these two components are disconnected.

Flexible-connected components, ${}_{F}\mathscr{C}$, $=_{df}$ subsets of system components that are path-connected between two other components not in the subsets.

$$ {}_{F}\mathscr{C} =_{df} \mathfrak{X} = \{\mathfrak{x} \mid \mathfrak{x} \in \mathfrak{S}_0 \wedge \exists \mathfrak{y}[(\mathfrak{x},\mathfrak{y}) \in_{c} \boldsymbol{E} \supset (\mathfrak{x},\mathfrak{y}) \in_{pc} \boldsymbol{E} \wedge \mathscr{F}(\mathfrak{X}_i)((\mathfrak{x},\mathfrak{y}))]\}; \text{ where} $$

$$ \mathscr{F}(\mathfrak{X}_i) = \{\mathfrak{X}_i \mid \mathfrak{X}_i \subset \mathfrak{S}_0 \wedge i>1 \wedge \forall \mathfrak{X}_i \exists \mathfrak{x} \in \mathfrak{S}_0 \exists \mathfrak{y} \in \mathfrak{S}_0[(\mathfrak{x},\mathfrak{y}) \in_{pc} \boldsymbol{E} \supset (\mathfrak{x},\mathfrak{X}_i),(\mathfrak{X}_i,\mathfrak{y}) \in_{pc} \boldsymbol{E}]\} $$

Flexible-connected components is defined as a set of components of the object-set; such that, the components are path-connected and are path-connected through two or more subsets of the object-set.

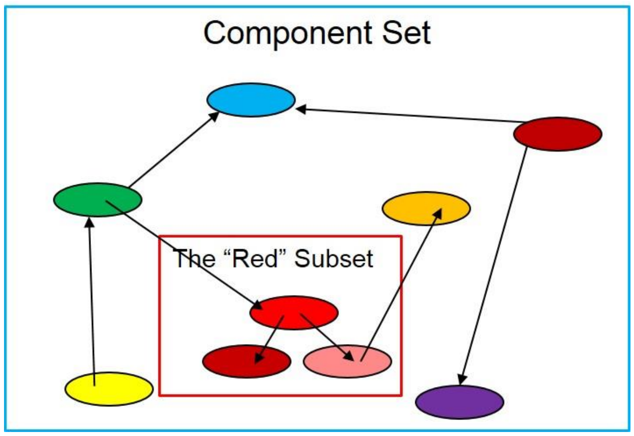
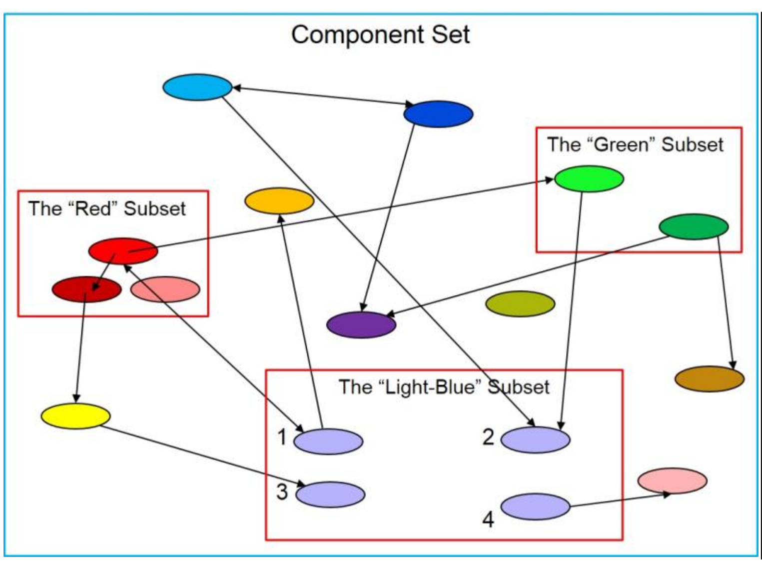


Figure 9. Diagram of a set of **flexible-connected components**. The Red Subset consists of three connected components. Green is path-connected to Red in this Red Subset, and Light-Red is connected to Orange. Therefore, Green and Orange of the Component Set are **flexible-connected** even though they are not connected directly within the Component Set outside of the Red Subset. As seen below, these flexible-connected components can be connected through a number of subsets, and the connections are a result of subset-connections rather than component-connections. Yellow is connected to Purple only through “Light-Blue Subset” → “Red Subset” → “Green Subset”. Yellow is connected to Pink through “Light-Blue Subset”.



The subsets are treated as “components” with respect to the components of the Component Set not in the subsets in terms of the affect relations. That is, a path-connection from the components outside the subsets to the subsets can be connected to any component of the subset, and then another component of the subset may be connected to a component outside the subset that results in the path-connection of those two components of the Component Set. For example, Yellow to Pink is “path-connected”. This is critical when; for example, the components of the subset are unknown, but we do know that there are path-connections to and from the subset.

Flexible affect relation, ${}_{F}\mathcal{A}$, $=_{Df}$ an affect relation comprised of flexible components.

$${}_{F}\mathcal{A} =_{df} \mathcal{A} \mid \forall(x,y)\in\mathcal{A}\,[x\in {}_{F}\mathcal{C}]$$

Flexible affect relation is defined as an affect relation; such that, all components are flexible-connected components.

Heterarchy, ${}_{HA}\mathcal{C}$, **or Network-connected components,** ${}_{NW}\mathcal{C}$, $=_{df}$ a set of system components that are non-hierarchical, path-connected components.

$${}_{NW}\mathcal{C} =_{df} \mathfrak{X} = \{x \mid x\in \mathfrak{S}_0 \wedge \exists y[(x,y)\in {}_{c}E \supset (x,y)\notin {}_{HO}\mathcal{C} \wedge (x,y)\in {}_{pc}E]\}$$

$${}_{HA}\mathcal{C} = {}_{NW}\mathcal{C}$$

Heterarchy is defined as a set of components of the object set; such that, if the components are connected, then they are not hierarchically connected and they are path-connected.

Figure 7 presents three component sets that represent a heterarchy.

Heterarchy affect relation, ${}_{HA}\mathcal{A}$, **or Network affect relation,** ${}_{NW}\mathcal{A}$, $=_{df}$ an affect relation comprised of network-connected components.

$${}_{NW}\mathcal{A} =_{df} \mathcal{A} \mid \forall(x,y)\in\mathcal{A}\,[x\in {}_{NW}\mathcal{C}]$$

$${}_{HA}\mathcal{A} = {}_{NW}\mathcal{A}$$

Heterarchy affect relation is defined as an affect relation; such that, all components are network-connected components.

Hierarchically-connected components, ${}_{HO}\mathcal{C}$, $=_{df}$ a set of system components that are unilaterally connected from a primary-initiating component.

$${}_{HO}\mathcal{C} =_{df} \mathfrak{X} = \{x \mid x\in\mathfrak{S}_0 \wedge \exists z\in\mathfrak{S}_0[(z\in {}_{pi}E \ \wedge (z,x)\in {}_{uc}E\ )) \wedge \forall y\in\mathfrak{S}_0((x,y)\in {}_{c}E :\supset: (x,y)\in {}_{uc}E \vee (z,y),(y,x)\in {}_{uc}E)]\}$$

Hierarchically-connected components is defined as a set of components of the object set; such that, the components are unilaterally-connected to other components from a primary-initiating component, and all connections are unilateral connections.

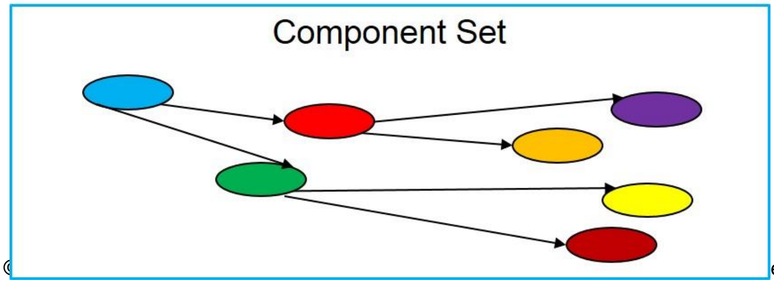


Figure 10. Diagram of a set of **hierarchically-connected components**. This hierarchy is from left-to-right.

Hierarchical ordered affect relation, $_{HO}\mathcal{A}$, $=_{df}$ an affect relation comprised of hierarchically connected components.

$$_{HO}\mathcal{A} =_{df} \mathcal{A} \mid \forall(x,y)\in\mathcal{A}\,[(x,y)\in_{c}\boldsymbol{E} \supset x\in_{HO}\mathcal{C}]$$

Hierarchical ordered affect relation is defined as an affect relation; such that, all components are hierarchically connected components.

Independent-connected components, $_{I}\mathcal{C}$, $=_{df}$ a set of system components that do not have connections to them.

$$_{I}\mathcal{C} =_{df} \mathfrak{X} = \{x \mid x\in \mathcal{S}_0 \wedge \exists y[(x,y)_{c}\boldsymbol{E} \supset x\in_{pi}\boldsymbol{E}]\}$$

Independent-connected components is defined as a set of components of the object set; such that, if the components are connected, then they are primary initiating.

Disconnected components are not considered as being "independent" in this definition. By the definition of the object-set, being derived from the relation-set, if a component is not related to any other component, then it is not part of the system, and, in fact, cannot be known.

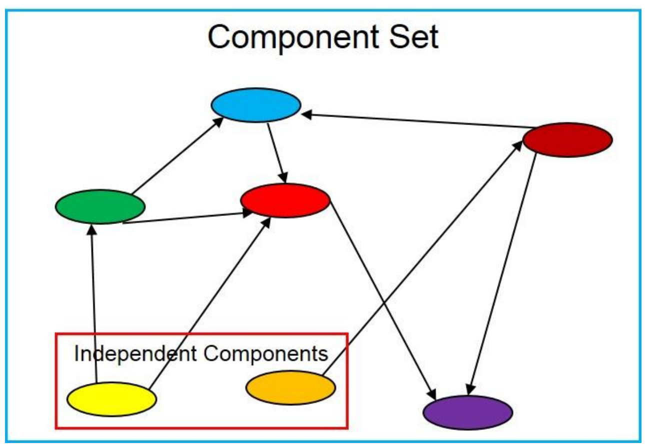


Figure 11. Diagram of a set of **independent-connected components**. That is, Yellow and Orange do not have any connections to them. The "Independent Components" Subset is an **independent affect relation** within the larger set. See definition below.

Independent affect relation, $_{I}\mathcal{A}$, $=_{df}$ an affect relation comprised of independent components.

$$_{I}\mathcal{A} =_{df} \mathcal{A} \mid \forall(x,y)\in\mathcal{A}\,[x\in_{I}\mathcal{C}]$$

Independent affect relation is defined as an affect relation; such that, all components are independent connected components.

Indirect-directed-connected components, $_{ID}\mathcal{C}$, $=_{df}$ a set of system components that have path-connections through other components.

$$_{ID}\mathcal{C} =_{df} \mathfrak{X} = \{x \mid x \in \mathbb{S}_0 \wedge \exists y[(x,y) \in {}_c E \supset (x,y)_{n>1}]\}$$

Indirect-directed-connected components is defined as a set of components of the object set; such that, if the components are connected, then the number of paths is greater than one.

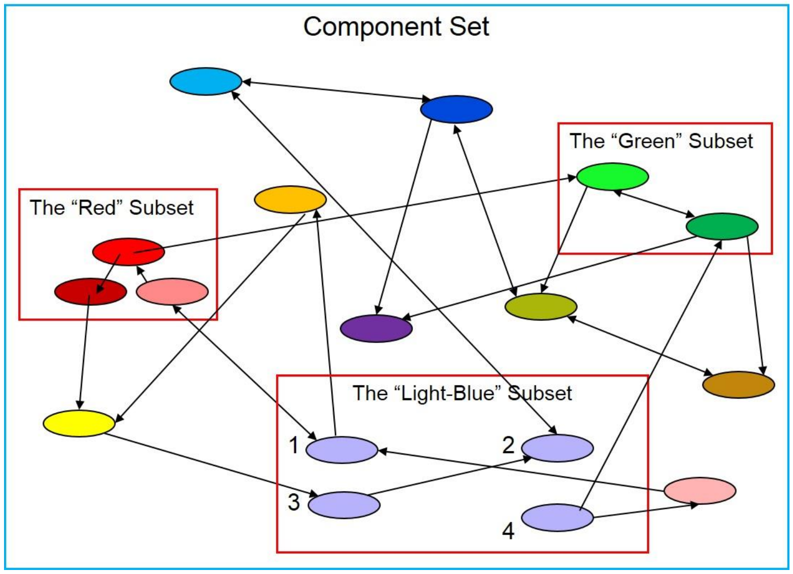


Figure 12. Diagram of a set of **indirect-directed connected components**. The components within the subsets do not have to be indirect-connected to any other components, since such subsets simply provide for **flexible-connected components**. It is only the components outside these subsets with which we are concerned.

Indirect-directed affect relation, $_{ID}\mathcal{A}$, $=_{df}$ an affect relation comprised of indirect-directed components.

$$_{ID}\mathcal{A} =_{df} \mathcal{A} \mid \forall(x,y) \in \mathcal{A}\, [x \in {}_{ID}\mathcal{C}]$$

Indirect-directed affect relation is defined as an affect relation; such that, all components are indirect-directed connected components.

Integrated-connected components (integration), $_{IG}\mathcal{C}$, $=_{df}$ maintenance of wholeness under change in system state.

$$_{IG}\mathcal{C} =_{df} \mathcal{M}([{}_W\mathcal{C}_{t(1)})] - [{}_W\mathcal{C}_{t(2)}]) < \alpha \mid \Delta\mathcal{S}_{t(1)\rightarrow t(2)}$$

Integrated-connected components is defined as a measure of the difference of wholly-connected components at times t(1) and t(2) are less than α, given a change in system state from time t(1) to time t(2).

Integration in SIGGS has been misidentified as being the result of a change in the environment, whereas the change actually occurs within the system; hence, a change in state.

Interdependent-connected components, ${}_{N}\mathscr{C}$, $=_{df}$ a set of system components that are completely connected.

$${}_{N}\mathscr{C} =_{df} \mathfrak{X} = \{\mathfrak{x} \mid \mathfrak{x} \in \mathfrak{S}_{O} \wedge \exists \mathfrak{y}[(\mathfrak{x},\mathfrak{y}) \in {}_{c}\boldsymbol{E} \supset (\mathfrak{x},\mathfrak{y}) \in {}_{cc}\boldsymbol{E}]\}$$

Interdependent-connected components is defined as a set of components of the object set; such that, if the components are connected, then they are completely connected.

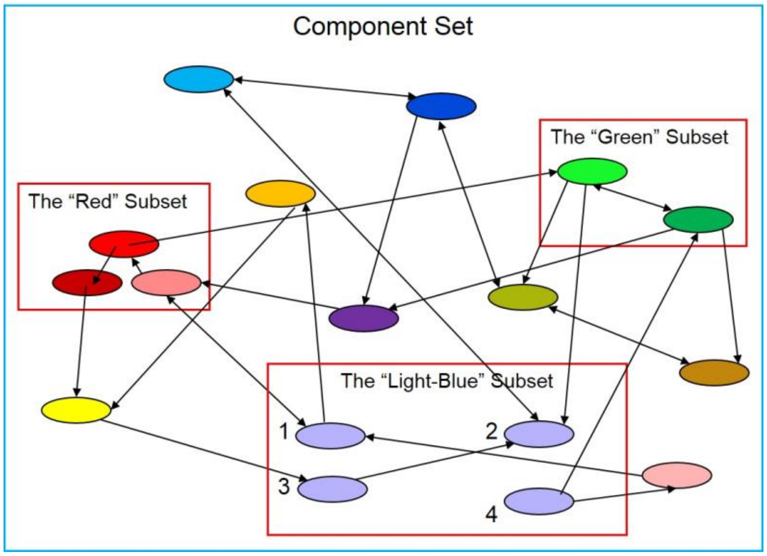


Figure 13. Diagram of a set of **interdependent-connected components**. The components within the subsets do not have to be interdependent-connected to any other components, since such subsets simply provide for **flexible-connected components**. It is only the components outside these subsets with which we are concerned. For those components, there is a path to and from each pair of components, possibly through one of the subsets.

Interdependent affect relation, ${}_{N}\mathcal{A}$, $=_{df}$ an affect relation comprised of interdependent components.

$${}_{N}\mathcal{A} =_{df} \mathcal{A} \mid \forall(\mathfrak{x},\mathfrak{y}) \in \mathcal{A}\,[\mathfrak{x} \in {}_{N}\mathscr{C}]$$

Interdependent affect relation is defined as an affect relation; such that, all components are interdependent connected components.

Network affect relation, ${}_{NW}\mathcal{A}$, **or Heterarchy affect relation,** ${}_{HA}\mathcal{A}$, $=_{df}$ an affect relation comprised of network-connected components.

$${}_{NW}\mathcal{A} =_{df} \mathcal{A} \mid \forall(\mathfrak{x},\mathfrak{y}) \in \mathcal{A}\,[\mathfrak{x} \in {}_{NW}\mathscr{C}]$$

$${}_{HA}\mathcal{A} = {}_{NW}\mathcal{A}$$

Network affect relation is defined as an affect relation; such that, all components are network-connected components.

Network-connected components, $_{NW}\mathcal{C}$, or Heterarchy, $_{HA}\mathcal{C}$, $=_{df}$ a set of system components that are non-hierarchical, path-connected components.

$$_{NW}\mathcal{C} =_{df} \mathfrak{X} = \{x \mid x \in \mathfrak{S}_0 \wedge \exists y[(x,y) \in_c \boldsymbol{E} \supset (x,y) \notin {}_{HO}\mathcal{C} \wedge (x,y) \in_{pc} \boldsymbol{E}]\}$$

$$_{HA}\mathcal{C} = {}_{NW}\mathcal{C}$$

Network-connected components is defined as a set of components of the object set; such that, if the components are connected, then they are not hierarchically connected and they are path-connected.

Segregation, $_{SG}\mathcal{C}$, $=_{df}$ Maintenance of independence under system environmental change.

$$_{SG}\mathcal{C} =_{df} \Delta\mathfrak{S}' \supset |[{}_I\mathcal{C}_{t(1)}] - [{}_I\mathcal{C}_{t(2)}]| < \alpha$$

Segregation is defined as a change in system environment implies that the change in independence is less than some value α.

Strongly-connected components, $_S\mathcal{C}$, $=_{df}$ a set of system components that are path-connected and there are at least two components that are path-connected in only one direction.

$$_S\mathcal{C} =_{df} \mathfrak{X} = \{x \mid x \in \mathfrak{S}_0 \wedge \exists y[(x,y) \in_c \boldsymbol{E} :\supset: (x,y) \in_{pc} \boldsymbol{E} \wedge \exists z \in \mathfrak{S}_0((x,z) \in_{uc} \boldsymbol{E})]\}$$

Strongly-connected components is defined as a set of components of the object set; such that, if the components are connected, then they are path-connected and there exists a component to which the components are unilaterally connected.

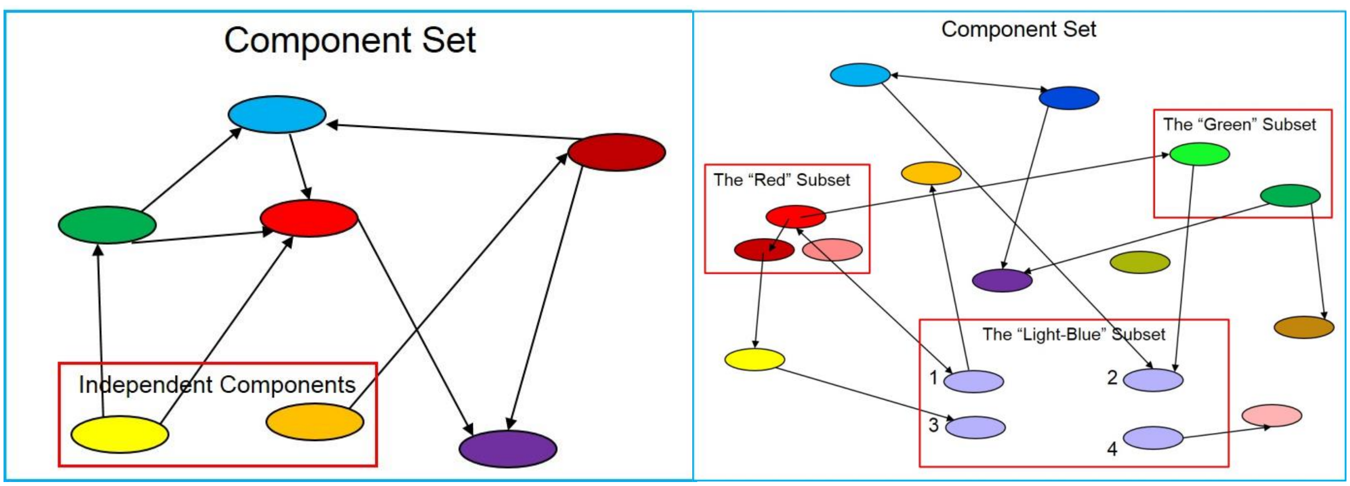


Figure 14. Diagram of a set of **strongly-connected components**. (1) All of the components in the left Component Set are pair-wise unilaterally connected in only one direction. (2) Yellow and Light Pink (right side) are unilaterally connected. (3) Blue and Purple are unilaterally connected. (4) Blue and Light Pink are unilaterally connected. These two Component Sets are **strong affect relations** as defined below.

Strong affect relation, ${}_S\mathcal{A}$, $=_{df}$ an affect relation comprised of strongly connected components.

$${}_S\mathcal{A} =_{df} \mathcal{A} \mid \forall(x,y)\in\mathcal{A}\,[x\in {}_S\mathcal{C}]$$

Strong affect relation is defined as an affect relation; such that, all components are strongly connected components.

Unilaterally-connected components, ${}_U\mathcal{C}$, $=_{df}$ a set of system components that are path-connected in one and only one direction.

$${}_U\mathcal{C} =_{df} \mathfrak{X} = \{x \mid x\in\mathfrak{S}_0 \wedge \exists y[(x,y)\in {}_c\boldsymbol{E} \supset (x,y)\in {}_{uc}\boldsymbol{E}]\}$$

Unilaterally connected components is defined as a set of components of the object set; such that, if the components are connected, then they are path-connected in only one direction.

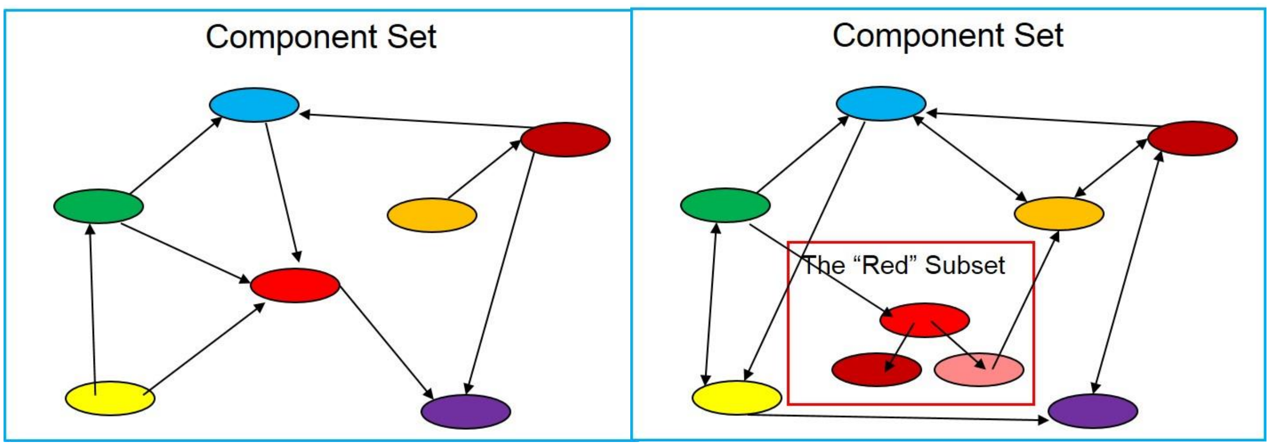


Figure 15. Diagram of sets of **unilaterally-connected components**. That is, all components in the left diagram above have connections that are path-connected in only one direction. The Component Set shown is a **unilateral affect relation** since all components are path-connected in only one direction. In the left diagram shown above, the subset is a **unilaterally-connected subset** and, therefore, a **unilateral affect relation**, but the other components of the Component Set are not.

Unilateral affect relation, ${}_U\mathcal{A}$, $=_{df}$ an affect relation comprised of unilaterally connected components.

$${}_U\mathcal{A} =_{df} \mathcal{A} \mid \forall(x,y)\in\mathcal{A}\,[x\in {}_U\mathcal{C}]$$

Unilateral affect relation is defined as an affect relation; such that, all components are unilaterally connected components.

Weakly-connected components, ${}_{WC}\mathscr{C}$, $=_{df}$ a set of system components that are path-connected, and there are at least two components that are not connected to each other.

$$ {}_{WC}\mathscr{C} =_{df} \mathfrak{X} = \{x \mid x \in \mathbb{S}_0 \wedge \exists y[(x,y) \in {}_{c}\boldsymbol{E} :\supset: (x,y) \in {}_{pc}\boldsymbol{E} \wedge \exists z \in \mathbb{S}_0((x,z) \in {}_{d}\boldsymbol{E})]\} $$

Weakly-connected components is defined as a set of components of the object set; such that, there are components that are connected components, implies that the components are path-connected and there are some components of the object set that are disconnected.

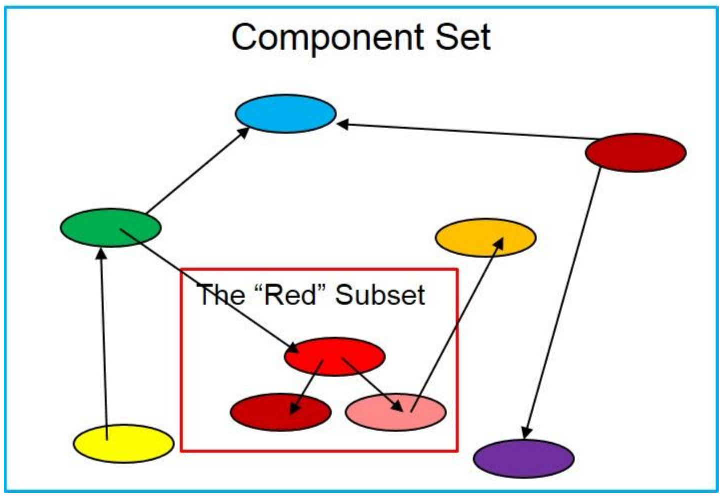


Figure 16. Diagram of a set of **weakly-connected components**.
That is; for example, Yellow and Purple are not connected. Further,
the following set is a **weak affect relation**: {Yellow, Purple}

Weak affect relation, ${}_{WC}\boldsymbol{\mathcal{A}}$, $=_{df}$ an affect relation comprised of weakly connected components.

$$ {}_{WC}\boldsymbol{\mathcal{A}} =_{df} \boldsymbol{\mathcal{A}} \mid \forall (x,y) \in \boldsymbol{\mathcal{A}}\, [x \in {}_{WC}\mathscr{C}] $$

Weak affect relation is defined as an affect relation; such that, all components are weakly connected components.

Wholly connected components, ${}_W\mathcal{C}$**,** $=_{df}$ a set of system components that are connected to all other components.

$$ {}_W\mathcal{C} =_{df} \mathfrak{X} = \{\mathfrak{x} \mid \mathfrak{x} \in \mathfrak{S}_0 \wedge \forall \mathfrak{y}[\mathfrak{y} \in \mathfrak{X} \supset \exists \mathfrak{x}((\mathfrak{x},\mathfrak{y}) \in {}_c\boldsymbol{E})]\} $$

Wholly connected components is defined as a set of components of the object set; such that, if a component is in the set, then there is another component of the set to which it is connected.

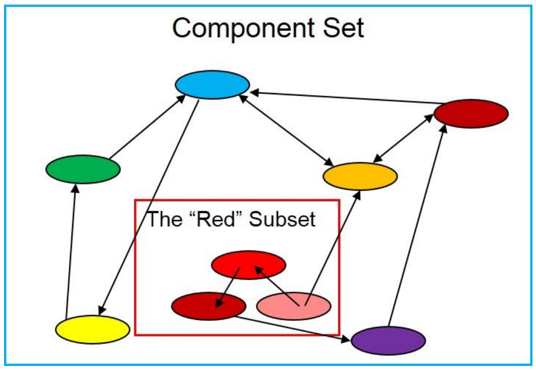


Figure 16. Diagram of a set of **wholly-connected components**. That is, the Red Subset is wholly-connected to all components outside the subset. However, the components outside the Red Subset are not **wholly-connected** to each other.

Whole affect relation, ${}_W\mathcal{A}$**,** $=_{df}$ an affect relation comprised of wholly connected components.

$$ {}_W\mathcal{A} =_{df} \mathcal{A} \mid \forall(\mathfrak{x},\mathfrak{y}) \in \mathcal{A}\, [\mathfrak{x} \in {}_W\mathcal{C}] $$

Whole affect relation is defined as an affect relation; such that, all components are wholly connected components.